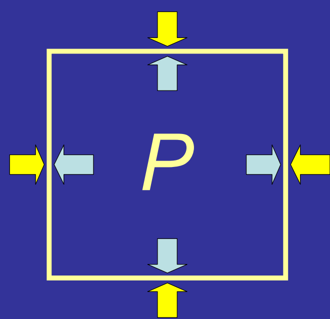
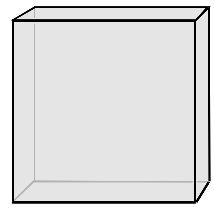
MET3003



Lecture 11: Pressure
26FEB07

To understand pressure we are back to thinking about ***Parcel***s of air



- The classic 1 m^3 cube
- Weighs about 1.2 kg at 1013 mb and 288K
- Density, ρ, as before is a function of "state variables".
  - Pressure, p,
  - Temperature, T, and
  - Water mixing ratio, q.
- Water influences density through
  - Weight of hydrometeors
  - Virtual temperature effects
- The p, T & q normally decrease with height

By way of review, we calculate density from the state variables with the Gas law for air

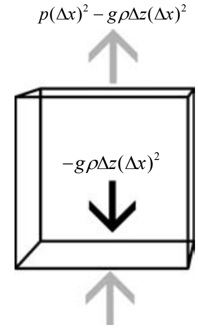
$$\frac{p}{\rho} = R_d T, \quad \text{or } \rho = \frac{P}{R_d T} = \frac{1013\text{ mb} \times 100\text{ Nt m}^{-2}\text{mb}^{-1}}{287\text{ m}^2\text{ s}^{-2}\text{ K}^{-1}\ 288\text{ K}}$$

Remember that $\text{Nt} = \text{kg m s}^{-2}$

So, $\rho = 1.226\text{ kg m}^{-3}$.

Note that denisty is proportional to p and inversely proportional to T.

Hydrostatic Pressure



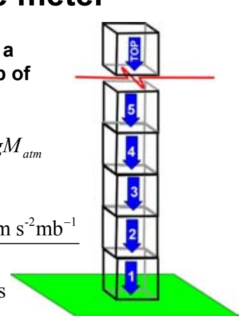
Pressure is the weight of air above. Taking the difference between pressure at top and bottom of the parcel:

$$\left(p_{\text{top}} - p_{\text{bottom}}\right)(\Delta x)^2 = -g\rho[\Delta z(\Delta x)^2]$$

$$\frac{p_{\text{top}} - p_{\text{bottom}}}{\Delta z} = -g\rho$$

Weight of the whole atmosphere above one square meter

Sum all the cubic meters in a column extending to the top of the atmosphere.



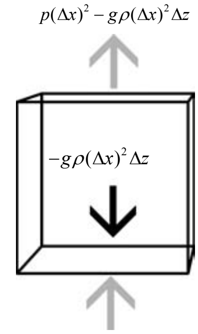
$$p_{sfc} = \sum_{\text{surface}}^{\text{top}} g\rho\Delta z = g\sum_{\text{surface}}^{\text{top}} \rho\Delta z = gM_{atm}$$

or

$$M_{atm} = \frac{p_{sfc}}{g} = \frac{1013\text{ mb} \times 100\text{ kg m s}^{-2}\text{mb}^{-1}}{9.8\text{ m s}^{-2}}$$

$$M_{atm} = 10337\text{ kg} \approx 10\text{ metric tons}$$

Hydrostatic Pressure and Density



Substituting from the gas law into the hydrostatic equation:

$$\frac{p_{\text{top}} - p_{\text{bottom}}}{\Delta z} = -g\rho = -\frac{gp}{R_d T},$$

$$\frac{1}{p}\frac{p_{\text{top}} - p_{\text{bottom}}}{\Delta z} = -\frac{g}{R_d T},$$

$H = RT_d / g$ is the "Scale Height."

Dimensions of length, i.e. m or km
Typically 6-8 km
Defines atmospheric vertical scale
Pressure can be written $p(z) = p(0)\exp\{-z/H\}$

Values of the Scale Height

Averaged over the depth of the troposphere the temperature is ½ (288.16 + 216.66) = 252.8 K.

$$H = \frac{R_d T}{g} = \frac{287\text{m}^2\text{s}^{-2}\text{K}^{-1} \times 252.8\text{K}}{9.8 \text{ m s}^{-2}} = 7406 \text{ m}$$

In the stratosphere below 20 km the temperature is constant at 216.66

$$H = \frac{R_d T}{g} = \frac{287\text{m}^2\text{s}^{-2}\text{K}^{-1} \times 216.66\text{K}}{9.8 \text{ m s}^{-2}} = 6345 \text{ m}$$

A Little Calculus Gives Us a Formula for Pressure as a Function of Altitude

Replacing the finite difference with a mathematical derivative:

$$\frac{1}{p}\frac{dp}{dz} = \frac{d \ln p}{dz} = -\frac{1}{H}$$

Integrating, $\ln p = -\frac{z}{H} + \text{constant}$

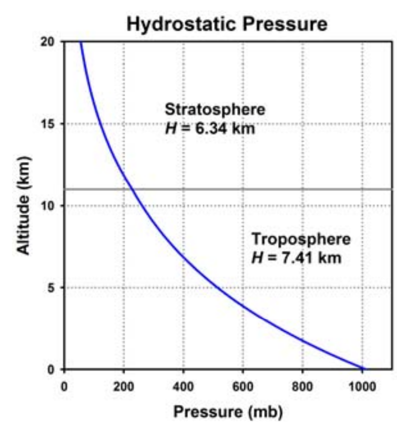
At the surface $z = 0$ and $p = p_s = 1013.6$ mb

So the constant of integration is $-\ln p_s$, and

$$p = p_s \exp\left\{-\frac{z}{H}\right\}$$

Examples

- At z = H = 7.4 km
  - $p = p_s e^{-1}$ = 1013.6 x 0.3687 =373 mb
- At z = 2H = 14.8 km
  - $p = p_s e^{-2}$ = 1013.6 x 0.135 =137 mb
- At z = 3H = 22.2 km
  - $p = p_s e^{-2}$ = 1013.6 x 0.049 =50.5 mb
- But actually the stratosphere is so much colder that we need to use a different value of *H*



Pressure as a Vertical Coordinate

- Meteorologists find it convenient to use the height where a fixed pressure is found rather than the pressure at a fixed height.

$$z = H \ln\frac{p_1}{p_2} = \frac{R_d T_v}{g} \ln\frac{p_1}{p_2}$$

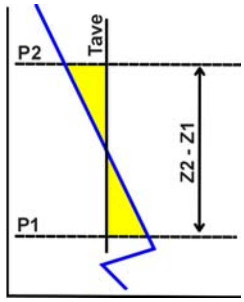
Called the ***Hypsometric Equation***
Note that the ***Thickness*** between the two pressure levels is proportional to the average virtual temperature of the layer between them

Altitudes of Some Standard Surfaces

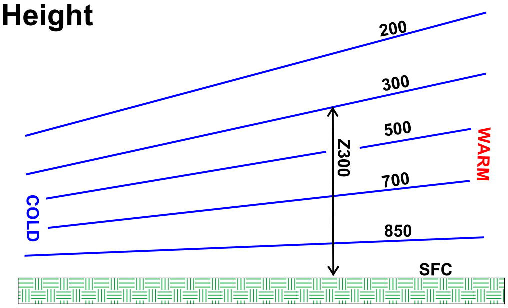
Pressure (mb)	Nominal Altitude (m)	Standard Atm. Altitude (m)
1013.6	0	0.0
1000	100	110.8
850	1500	1457.7
700	3000	3022.6
500	5000	5573.8
300	9000	9164.0
200	12000	11798.8

Using Layer-Mean Temperature to Calculate Thickness

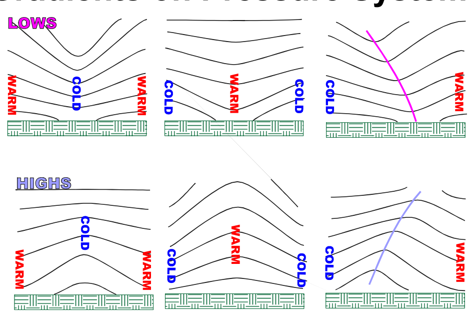
- Divide the sounding into layers
  - 1000 to 850
  - 850 to 700
  - etc.
- Determine the layer-mean virtual temperature as indicated.
- Use the hypsometric equation to compute the layer thicknesses
- Add them up to get heights



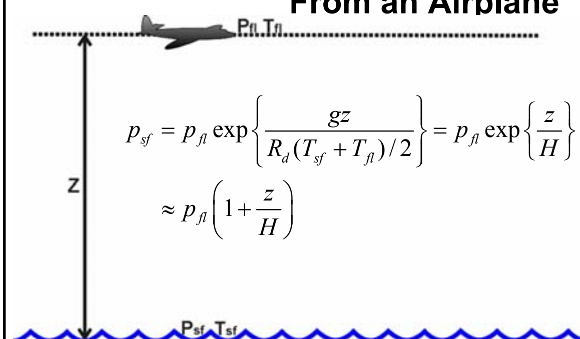
Horizontal Temperature Gradients Cause Gradients of Thickness and Height



Effect of Horizontal Temperature Gradients on Pressure Systems



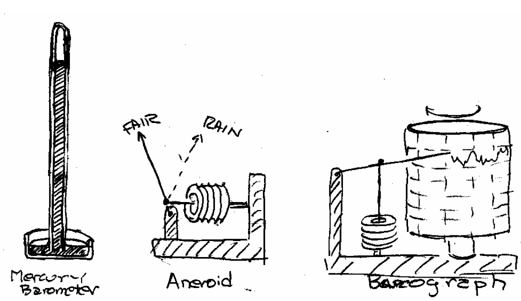
Calculating Surface Pressure From an Airplane



$$p_{sf} = p_{fl} \exp\left\{\frac{gz}{R_d (T_{sf} + T_{fl})/2}\right\} = p_{fl} \exp\left\{\frac{z}{H}\right\}$$

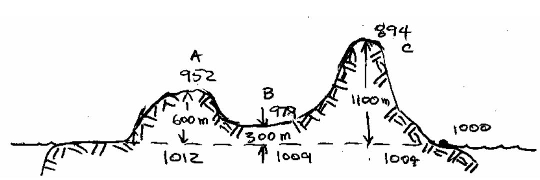
$$\approx p_{fl}\left(1 + \frac{z}{H}\right)$$

Measuring Pressure



Station and Sea-Level Pressure

- Station pressure: Actually read from the barometer
- Sea-level pressure: Corrected for station elevation
  - About 1 mb for every 10 m of altitude
  - Plotted to make isobars on weather maps



Summary

- Understand and be able to use the ***Hydrostatic Law***
- Be able to "weigh" the atmosphere knowing the surface pressure
- Understand the concept of ***Scale Height*** and be able to use it to calculate vertical pressure variation
- Understand ***Isobaric***, or ***Pressure, Coordinates***
- Know what the ***Hypsometric Equation*** is and how it is used
- Understand and be able to draw and explain how horizontal temperature gradients
  - Lead to height gradients
  - Affect the vertical structure of high and low pressure systems
- Know what instruments are used to measure pressure
- Know the difference between ***Station Pressure*** and ***Sea-Level Pressure*** and know how the latter ic calculated from the former