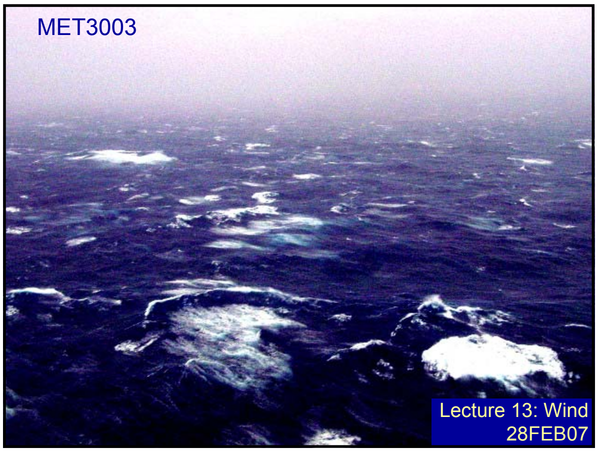
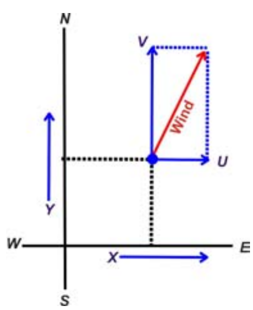



# Dynamics: The study of atmospheric motions

- ***Wind***: Air in motion relative to the surface of the Earth. ---*Glossary of Meteorology*
- **Newton's Laws of Motion:**
  1. Every object continues in a state of rest or straight-line motion at a fixed speed unless some force act upon it.
  2. The rate of change of velocity (speed and/or direction) is proportional to the force imposed divided by the mass of the object. ($F = Ma$)
  3. For every action there is an equal an opposite reaction.

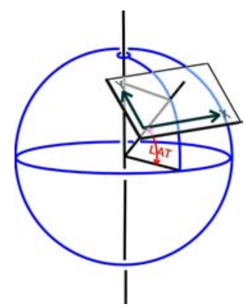
# Meteorological Coordinates

- Cartesian coordinates
  - Y points North
  - X points East
  - South and west are negative
- Positions and winds are vectors: magnitude & direction
- Winds
  - Southerly (from the S) winds are positive
  - Westerly (from the W) winds are positive
  - Northerly and Easterly winds are negative



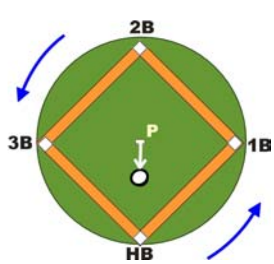

# Tangent Plane Geometry

- Ordinary Cartesian coordinates tangent to the globe as a selected point
- Pretends that the Earth is locally flat.
- Works OK for understanding, but…

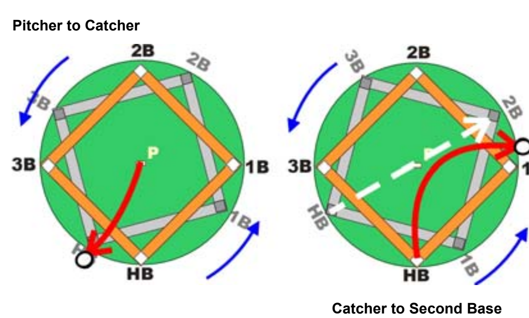



# **Forces Due to the Earth**

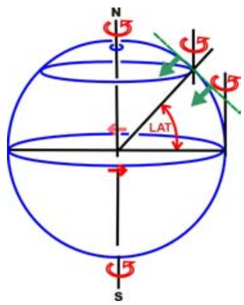
- Gravity
  - Pulls all objects toward the center of the Earth
  - Acceleration is ~9.8 m s$^{-2}$
- Centripetal acceleration due to Earth's rotation
  - Small
  - Generally combined with gravity
- Coriolis force
  - Also due to Earth's rotation
  - Affects only moving objects
  - Deflects to the right in the Northern hemisphere
  - To the left in the Southern Hemisphere
- To understand the Coriolis force consider infield throwing practice on a rotating baseball diamond




# Coriolis Force in Action

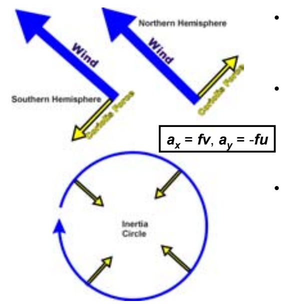


## Coriolis Force on a Spherical Earth




- Earth rotates 2π radians in 24 h, or in 86400 s
- Coriolis force acts is a plane perpendicular to the axis of rotation.
- But the local tangent plane makes an acute angle with the axis
- Coriolis force is always much weaker than gravity
- So we see only the part that acts in the local horizontal
- This effective rotation rate turns out to be proportional to twice the planet's angular velocity times the sine of latitude
- Or f = (4π/86400 s) sin φ = 1.454 x $10^{-4}$ $s^{-1}$ sin φ = 2Ω sin φ
- Called the ***Coriolis Parameter***

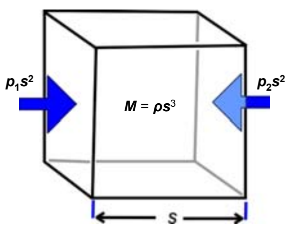
## Coriolis Force




$a_x = fv$, $a_y = -fu$

- Values of the Coriolis Parameter
  - At 45° Latitude = 1.0 x $10^{-4}$
  - At 20° Latitude = 0.5 x $10^{-4}$
- The Coriolis force always acts perpendicular to the wind
  - To the right in the Northern Hemisphere
  - To the left in the Southern Hemisphere
- Can cause air to move in circular orbits
  - Called "***Inertia Circles***"
  - Period is 2π/f
  - Always rotate anticylonically
  - Period is 24 h at 30° latitude
  - Longer equatorward and shorter poleward

## Pressure Gradient Force



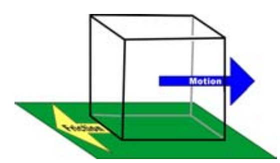

$$a = \frac{F}{M} = \frac{p_2 s^2 - p_1 s^2}{\rho s^3}$$

$$= \frac{1}{\rho}\frac{p_2 - p_1}{s} = \frac{1}{\rho}\frac{\Delta p}{\Delta x} = \frac{1}{\rho}\frac{\partial p}{\partial x}$$

- Horizontal gradients of pressure cause net forces to act on air parcels
  - $p_1$ pushes from the left
  - $p_2$ pushes from the right
  - If one is greater than the other, the parcel accelerates toward low pressure
- Since the net force is pressure times area and the mass is density times volume…
  - The magnitude of the acceleration is proportional to the gradient of pressure

## Friction

- Important only near the surface
- Always acts to slow the wind
- Simple approximation:
  - Proportional to the wind speed
  - Acts in the opposite direction
- $a_x = -ku$ and $a_y = -kv$



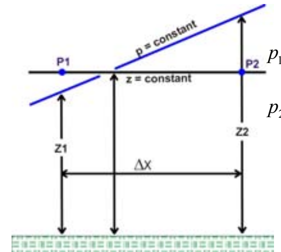

## Equations of Motion in Height Coordinates

$$\frac{du}{dt} - fv = -\frac{1}{\rho}\frac{\Delta p}{\Delta x} - ku$$

$$\frac{dv}{dt} + fu = -\frac{1}{\rho}\frac{\Delta p}{\Delta y} - kv$$

Note: du/dt and dv/dt are derivatives of u & v --- their accelerations

## Pressure as a Vertical Coordinate




$$p_1 = p - \frac{\Delta p}{\Delta z}(z_1 - z) = p + g\rho(z_1 - z)$$

$$p_2 = p - \frac{\Delta p}{\Delta z}(z_2 - z) = p + g\rho(z_2 - z)$$

$$\frac{\Delta p}{\Delta x} = \frac{p_2 - p_1}{\Delta x} = \frac{g\rho(z_2 - z_1)}{\Delta x}$$

$$\text{or} \quad \frac{1}{\rho}\frac{\Delta p}{\Delta x}_{z=\text{const}} = g\frac{\Delta z}{\Delta x}_{p=\text{const}}$$

## Equations of Motion in Pressure Coordinates

$$\frac{du}{dt} - fv = -g\frac{\Delta z_p}{\Delta x} - ku$$

$$\frac{dv}{dt} + fu = -g\frac{\Delta z_p}{\Delta y} - kv$$

## Terms in the Equations of Motion

- Neglect Friction
- Wind is blowing 10 m $s^{-1}$
- It takes about a day (86400s ~ $10^5$ s) for the wind to change significantly
- Acceleration 10 m $s^{-1}$/($10^5$ s) = 1 x $10^{-4}$ m $s^{-1}$
- At 45° latitude $f = 10^{-4}$ $s^{-1}$
- Coriolis term is $10^{-4}$ x 10 = 1 x $10^{-3}$ m $s^{-2}$
- Pressure gradient term is 10 mb in 1000 km = (10 mb x 100 kg $m^{-1}$ $s^{-2}$ /mb)/(1 kg $m^{-3}$ x $10^6$ m) = $10^{-3}$ m $s^{-1}$
- So the Coriolis and pressure-gradient terms are about the same size.
- While the acceleration term is a factor of ten smaller.
- ***Geostrophic Balance***

$$\frac{du}{dt} - fv = -\frac{1}{\rho}\frac{\Delta p}{\Delta x}$$

$$\frac{dv}{dt} + fu = -\frac{1}{\rho}\frac{\Delta p}{\Delta y}$$

## Geostrophic Balance

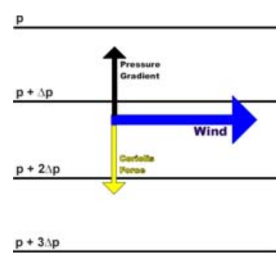
$$v = \frac{g}{f}\frac{\Delta z_p}{\Delta x}, \quad u = -\frac{g}{f}\frac{\Delta z_p}{\Delta y}, \text{ or}$$

$$v = \frac{1}{f\rho}\frac{\Delta p}{\Delta x}, \quad u = -\frac{1}{f\rho}\frac{\Delta p}{\Delta y}$$

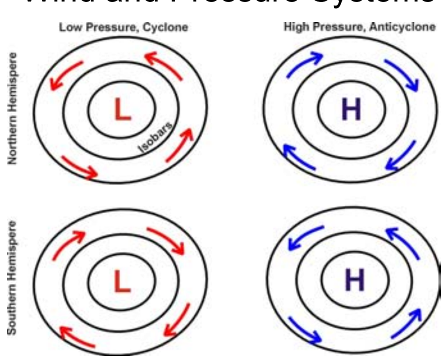
Example: There is a N-S pressure difference of 12 mb between two points 1000 km apart at 40° latitude. Air density is 1.2 kg m-3. What is the direction and speed of the Geostrophic wind?

## Balance of Forces

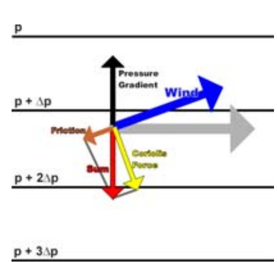
- Pressure gradient pushes toward lo pressure
- Coriolis force pushes back toward high pressure
- Wind deflects to right of pressure gradient
- LAW of STORMS: If the wind is blowing from left-to-right you are facing toward low pressure in the Northern Hemisphere.
- It's right-to-left in the Southern Hemisphere




## Wind and Pressure Systems



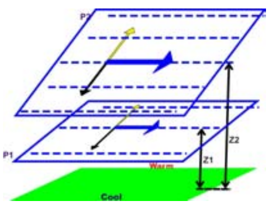

## Effect of Friction




- Friction slows the wind.
- The pressure gradient is then stronger than the Coriolis force
- Air starts to accelerate toward low pressure
- Until the combined effects of friction and the new Coriolis force balance the pressure gradient

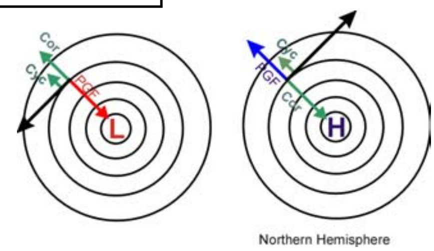
# Thermal Wind

- As we saw last time, temperature gradients lead to changes in isobaric height gradients with altitude.
- This produces changes in geostropic wind.
- Proportional to horizontal gradients of virtual temperature.
- Geostrophic wind aloft minus that at lower levels is called the **Thermal Wind**
- It's relationship to temperature field is like the relationship between the geostrophic wind and pressure




# Gradient Wind Equation

$$\boxed{\frac{v^2}{r} + fv = g\frac{\Delta\phi}{\Delta r}}$$




# Summary

- Know and be able to apply Newton's laws of motion
- Understand how meteorological coordinates are used in a plane tangent to the spherical Earth
- Be able to explain and use the ***Coriolis Force***
- Be able to explain and use the ***Pressure Gradient Force***
- Be able to explain and use the surface ***Frictional Force***
- Understand how these forces combine to produce the meteorological momentum equations
- Understand the scaling argument for prevalence of ***Geostrophic Balance*** in the Atmosphere and the balance of forces in geostrophic flow
- Be able to calculate the geostrophic wind from a specified pressure distribution
- Be albe to draw and explain wind circulations around centers of high and low pressure in the Northern and Southern Hemispheres
- Be able to explain the effect of friction on the wind
- Be able to explain and interpret ***Thermal Wind*** balance
- Be able to explain and interpret ***Gradient-Wind*** balance
- **Reading for next time: Chapter 7 Circulation of the Atmospheric**