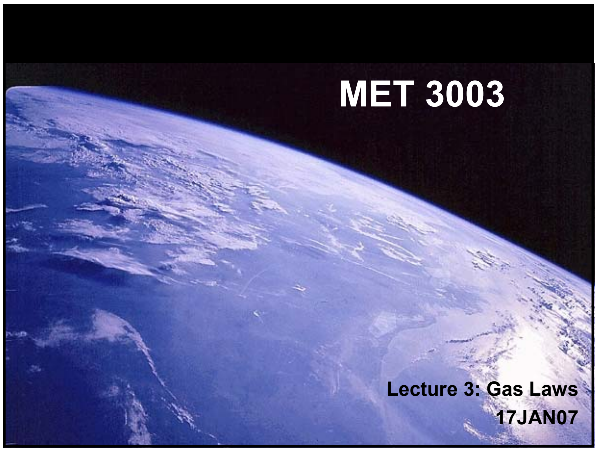
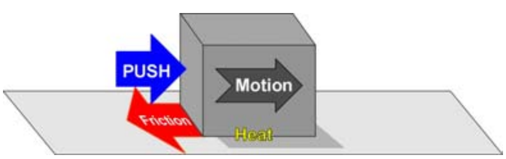


Force and Energy

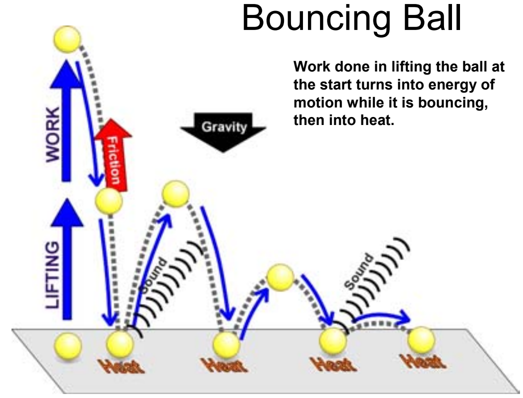
- Newton's second law of motion: Force is equal to mass times acceleration.
- Acceleration is the change in velocity over time.
  - Units of velocity--- m s^{-1}, Units of acceleration--- m s^{-2}
  - Units of force ---- kg m s^{-2} = Nt = Newton
- Definition: Work (change in mechanical energy) is force acting through a distance.
- Mechanical energy can be converted to heat.
  - Units of mechanical energy = kg m^2 s^{-2}= J = Joule
  - NOTE: Energy per unit mass has units of velocity^2 = m^2 s^{-2}
  - Heat energy is often measured in calories = 4.184 J
  - But we will use Joules throughout
- Although energy cannot be created or destroyed, it does change form (i.e. mechanical to heat) all the time

Work, Energy & Heat

- Pushing the block across the table does work
  - Force acting through a distance
- Friction does work
- Energy realized through work turns into heat in the block and table
- That is, to motions of molecules, not of the block
- What if there were no friction?

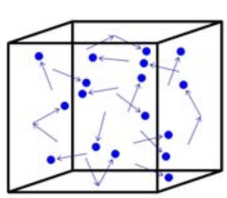


Bouncing Ball



A Cubic Meter of Air

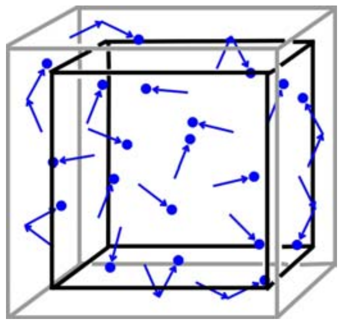
- Volume = *V*, containing *N* molecules
- At temperature *T* and pressure *p*.
- Pressure is force per unit area = Nt m^{-2} = Pascals
- *Absolute* temperature *T* K = *T*ºC + 273.16
- Individual molecules move because the temperature is above 0 K.
- Molecules bump into the walls causing pressure
- Energy of the average, individual molecule's motion is proportional to the temperature
  - $E = kT$ (k = 1.38 x10^{-23} J K^{-1} Boltzmann's constant)



Gas Laws

- Raising the temperature or adding more molecules will increase the pressure if the volume stays the same.
- OR … Raising the temperature or adding more molecules will increase the volume if the pressure stays the same.
- ***Boyles' Law***:
  - For a fixed mass of gas, the volume is inversely proportional to the pressure if the temperature is constant.
- ***Charles' Laws***:
  - For a fixed mass of gas, the volume is proportional to the (absolute) temperature if the pressure is constant.
  - For a fixed mass of gas, the pressure is proportional to the (absolute) temperature if the volume is constant.

Charles' Laws



The Equation (of state) that Relates Temperature (*T*), Pressure (*p*), number of kmoles (*n*) and Volume (*V*) is:

$$pV = nR^*T$$

*R** is fudge factor, called the **universal gas constant.**

It is equal to 8314.3 J deg^{-1} $kmole^{-1}$

What is all this "kmole" stuff?

- A "kmole" for a given substance is just its molecular weight in kilograms.
- Molecular weight = sum of the number of protons and neutrons in the *molecule* plus some slop because of binding energy and isotope mixtures. For example:
  - N_2 = 14 + 14 = 28, O_2 = 16 + 16 = 32,
  - O_3 = 16 + 16 +16 = 48, Ar = 40
  - CO_2 = 14 + 16 + 16 = 46, H_2O= 1 + 1 + 16 = 18
- It represents Avogadro's number, N_A=6.022 x10^{26} , of atoms per kmole.
- The universal gas constant is the product of Boltzman's constant with Avagadro's numero k x N_A

Air is a Mixture of Gasses

- M_d , the average molecular weight of dry air
  - Is a bit more than 28 (Actually 28.97) since air is mostly N_2 with about 20% O_2 . Rewriting the gas law for a **mass *m* of dry air**
- $R_d = R^*/M_d$ is the **gas constant for dry air**, = 8314.3/28.97 = 287 J deg^{-1} kg^{-1} = 287 (kg m^2 s^{-2}) deg^{-1} kg^{-1} = 287 m^2 s^{-2} deg^{-1}

$$pV = \frac{m}{M_d} N_A kT = \frac{m}{M_d} R^* T = mR_d T$$

Dalton's Law

- Each gas in a mixture exerts a partial pressure equal to the pressure that it would exert if it were by itself.

$$PV = (P_1 + P_2 + P_3 + \ldots)V = R^*\left(\frac{m_1}{M_1} + \frac{m_2}{M_2} + \frac{m_3}{M_3} + \ldots\right)T = \frac{R^* m_d T}{M_d}$$

Where:

$$\frac{1}{M_d} = \frac{1}{m_d}\left(\frac{m_1}{M_1} + \frac{m_2}{M_2} + \frac{m_3}{M_3} + \ldots\right), \text{ and } m_d = m_1 + m_2 + m_3 + \ldots$$

Density of Air

- The density of a gas is defined as m/V so that the gas law can be written:

$$\frac{p}{\rho} = p\alpha = R_d T,$$

where $\rho = m_d / V$ is the density, and

α is the "specific volume" = $1/\rho$

Calculation of Density

- Pressure is measured in millibars (mb), or equivalently, hPa = 100 Pascals
  - 1 mb = 100 Nt m^{-2} = 100 kg m^{-1} s^{-2}
- Calculate density of dry air at 1000 mb and 20°C

  ρ = (1000 x 100 kg m^{-1} s^{-2})/[287 m^2 s^{-2} deg^{-1} x (273.16 + 20 deg)]

  = 100000/(287x293.16) = 1.19 kg m^{-3}

Let's apply the gas law to water vapor

$$e\alpha_v = \frac{R^*}{M_v} T = R_v T$$

where $R_v = R^* / M_v = 461 = \mathrm{m}^2\ \mathrm{s}^{-2}\ \mathrm{deg}^{-1}$.

- The gas constant for dry air is 287 m^2 s^2
- The ratio of the gas constant for dry air divided by the gas constant for water vapor is 0.622
- Why is the dry-air constant so much less?
- Given that water vapor can be 4% of the air, is it an important factor in density?

Virtual Temperature

Density of a mixture of dry air and water vapor

$$\rho = \frac{m_d + m_v}{V} = \rho_d + \rho_v,$$

If we know the "partial pressures" of dry air and water vapor, we can calculate this density:

$$e = R_v \rho_v T \quad \text{and} \quad p_d = R_d \rho_d T, \quad \text{where} \quad p = p_d + e$$

So the density of the mixture is:

$$\rho = \frac{p-e}{R_d T} + \frac{e}{R_v T} = \frac{p}{R_d T}\left[1 - \frac{e}{p}\left(1 - \frac{R_d}{R_v}\right)\right]$$

$$= \frac{p}{R_d T}\left[1 - \frac{e}{p}(1 - 0.622)\right]$$

Which we can rewrite as,

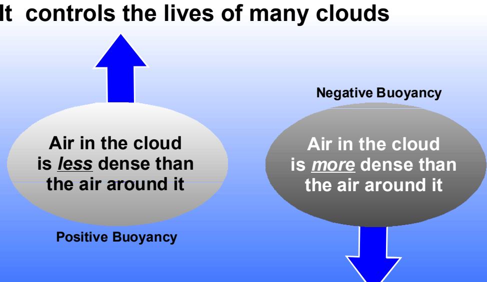
$$\rho = \frac{p}{R_d T_v}, \quad \text{where} \quad T_v = \frac{T}{1 - 0.378\frac{e}{p}}$$

is the "virtual" temperature.

Why do we have this morbid preoccupation with density?

Buoyancy depends upon the density difference between inside and outside a cloud

It controls the lives of many clouds



Summary

- Understand the definitions of and relationships among **force**, **energy**, **work**, **heat** and **pressure**
- Understand the universal gas law and its application to dry air and water vapor
- Be able to calculate density of dry air from temperature and pressure
- Understand the effect of water vapor on density
- Know the definition of **Virtual Temperature** and why it is used
- Be thinking about buoyancy since we will encounter it again
- Read Chapter 3 in *L&T* for next week